Solutions Excercises Week 4

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Exercise 1.

a)

The quantum dot is excited with a repetition period of 6 ns like we see in Fig. 1. The photoluminescence of course has the same periodicity like we see in Fig. 2. The decay is not a simple exponential decay but has two components.

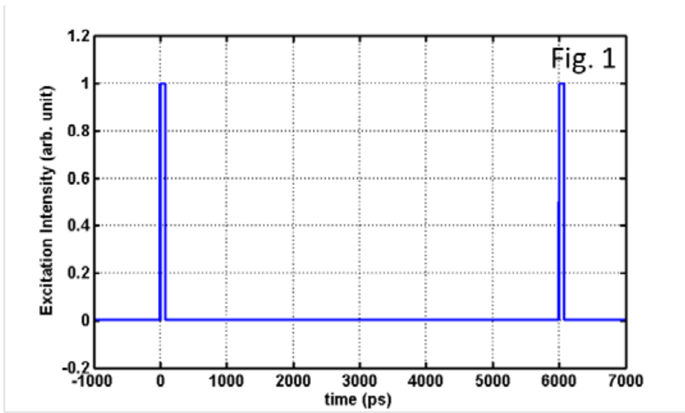
$$I = I_1 e^{-t/\tau_1} \quad \text{for } 0 < t < 2300\,ps$$

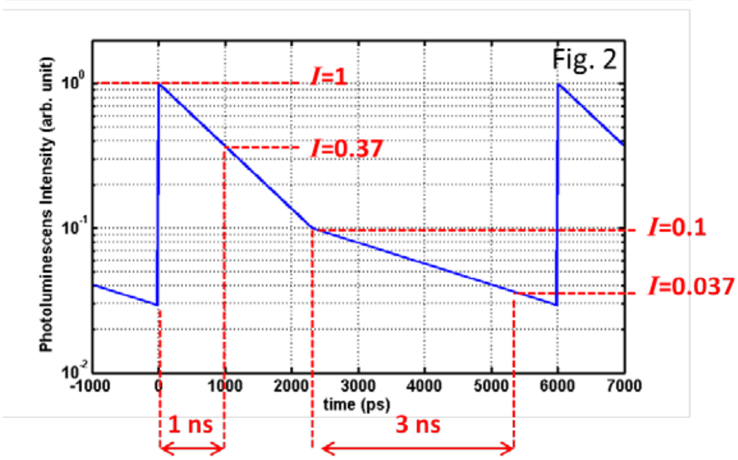
$$I = I_2 e^{-t/\tau_2} \quad \text{for } 2300 < t < 6000\,ps$$

In the time period $t = 0 \; to \; t = \tau_1$ the intensity falls from 1 to $e^{-\tau_1/\tau_1} = e^{-1} = 0.37$

The decay time is thus $\tau_1 = 1\,ns$. See Fig. 2

In the same way $\tau_2 = 3\,ns$. See Fig. 2





b)
The fastest measured component of the decay was roughly 1 ns. Now we must check how fast our diode is. If the response time is on the same time scale there is a chance that the measured decay is erroneous. The real decay could be much faster, it is just that we can't measure it.

The response time of a pin photodiode is limited by the RC constant and the drift time. We can ignore the diffusion time since most of the absorption in a pin diode happens in the intrinsic region where we have an electric field separating the charge carriers quickly.

RC
The pin diode can be approximated by a plate capacitor (see Fig 3)

$$C = \varepsilon_r \varepsilon_0 \frac{A}{W} = 11.9 * 8.85 * 10^{-12} * \frac{\pi(0.5 * 10^{-3})^2}{25 * 10^{-6}} = 3.3\,pF$$

Where A is the area of the device, W is the length of the intrinsic region.
ε_r and ε_0 are the relative and vacuum permittivity. ε_r for Si is found among the material parameters given in the question. ε_0 can be found in any formula book or at the back cover of "Fundamentals of Photonics".

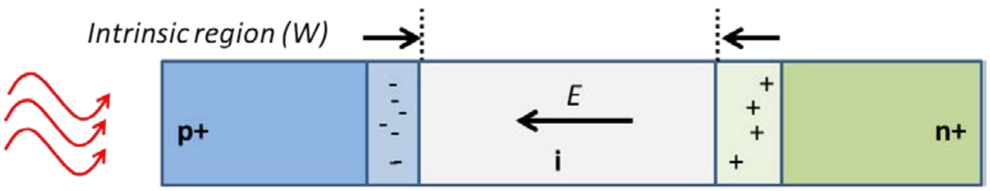


Fig. 3

The RC constant is thus $RC = 50\Omega * 3.3\,pF = 50 * 3.3 * 10^{-12} = 170\,ps$

Drift time
To calculate the drift time you need to know the electric field. You can then calculate it from the mobility or get it from a graph like the one given in the exercise. Note that the drift velocity saturates at high fields.

$$F = \frac{5\,V}{25\,\mu m} = 2 * 10^5\,V/m = 2 * 10^3\,V/cm$$

[From drift velocity chart, assuming the same drift velocity for electrons and holes, assuming T=300K]

- ⇨ $v_{drift} = 3 * 10^6 cm/s = 3 * 10^4 m/s$
- ⇨ $\tau_{drift} = \frac{W}{v_{drift}} = \frac{25*10^{-6}}{3*10^4} = 830\,ps$

Ans: The RC constant is fine but the drift time is quite close to the fastest decay time 1 ns. It is likely that the measured decay time is affected by the slow response of the photodiode.

c)
The drift velocity 3*10^4 m/s. There is still room to improve it by a factor 3 by increasing reverse bias and thereby reducing the drift time.

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$$\Re = G\eta \frac{\lambda[\mu m]}{1.24}$$

We are here working with a detector with ideal quantum efficiency, $\eta = 1$, and unity gain, $G = 1$.

The ideal responsivity will then rise linearly up to the bandgap wavelength, λ_g. See red trace in Fig. 4

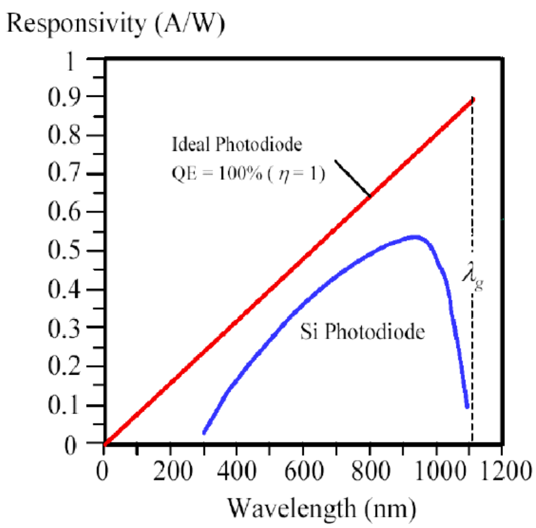

Fig. 4

a) $\lambda_g(Si) = 1.11\mu m \;\Rightarrow\; \Re_{max} = \frac{1.11}{1.24} = 0.90$ A/W

b) $\lambda_g(GaAs) = 0.873\mu m \;\Rightarrow\; \Re_{max} = \frac{0.873}{1.24} = 0.70$ A/W

c) $\lambda_g(InSb) = 7.29\mu m \;\Rightarrow\; \Re_{max} = \frac{7.29}{1.24} = 5.9$ A/W

Note that this is a simplified picture. In reality the maximum responsivity lies at shorter wavelength than the bandgap. This is because close to the bandgap the absorption coefficient is quite small giving a reduced η. Compare the blue and red trace in Fig. 4.

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At steady state $\Delta n = R\tau$

Where R is the generation rate and τ is the recombination lifetime.

Calculate R:
PD = power density
R= generated carriers/s=$PD * \eta/(hc/\lambda)$

Calculate photogenerated carrier concentration:

$$\Delta n = R\tau = \frac{PD * \eta * \lambda * \tau}{hc} = \frac{10^{-3} * 0.5 * 10^{-6} * 10 * 10^{-6}}{hc} = 2.5 * 10^{10}\ cm^{-3}$$

$$\Rightarrow \frac{\Delta n}{n_i} = \frac{2.5*10^{10}}{1.5*10^{10}} = 167\%$$

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Calculate quantum efficiency:

$$\eta = \frac{collected\ electrons}{incoming\ photons} = \frac{2 * 10^{12}}{6 * 10^{12}} = 1/3$$

Calculate quantum responsivity:

$$\Re = \eta \frac{\lambda[\mu m]}{1.24} = \frac{1}{3} * \frac{1.55}{1.24} = 0.42\ A/W$$

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Calculate quantum efficiency:

$$\Re = G\eta \frac{\lambda[\mu m]}{1.24} \Rightarrow$$

$$\Rightarrow\ \eta = \frac{\Re * 1.24}{G * \lambda[\mu m]} = \frac{12 * 1.24}{20 * 1.55} = 0.48$$

Calculate photocurrent:

$$I = \eta G e \Phi = 0.48 * 20 * 1.6 * 10^{-19} * 10^{10} = 15\ nA$$

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The photogenerated current is

$$\bar{\imath} = eG\eta\phi = e * 100 * 0.8 * \frac{10 * 10^{-9}}{\left(\frac{1240}{870}\right) * e} = 560\, nA$$

The variance of this current (arising from photon shot noise, gain and the excess noise factor) is

$$\sigma_i{}^2 = 2eB\bar{\imath}GF$$

So we need to calculate the excess noise factor, *F*, which depends on the gain and on the ionization ratio. If *k* is close to 1 both electrons and holes contribute to gain via impact ionization. This means that the avalance doesn't just go in one direction.

$$F = k * \bar{G} + (1-k)\left(2 - \frac{1}{\bar{G}}\right) = 51$$

$$\Rightarrow$$

$$\sigma_i{}^2 = 2eB\bar{\imath}GF = 2e * 100 * 10^3 * \bar{\imath} * 100 * 51 \;\Rightarrow\; \sigma_i = 9.6\, nA$$

The dark current has the same shot noise variance as the photo generated current. Again the variance is increased by the excess noise factor

$$\bar{\imath}_{Dark} = 1\, nA$$

$$\sigma_{iDark}{}^2 = 2eB\bar{\imath}_{Dark}GF = 2e * 100 * 10^3 * \bar{\imath}_{Dark} * 100 * 51 \;\Rightarrow\; \sigma_{iDark} = 0.40\, nA$$

Finally we have Johnson-Nyquist noise due to thermal motion of carriers in the leads.

$$\sigma_{JN}{}^2 = \frac{4k_BTB}{R} = \frac{4k_B * 300 * 100 * 10^3}{10^3} \Rightarrow \;\sigma_{JN} = 1.3\, nA$$

We see that the photocurrent noise dominates over the two other terms so we are in the photon shot noise limited regime.

The total signal to noise ratio is:

$$SNR = \frac{\bar{\imath}^2}{\sigma_i{}^2 + \sigma_{iDark}{}^2 + \sigma_{JN}{}^2} = 3400$$

Answer: The total SNR is 3400 based on
Photocurrent RMS= 9.6 nA
Dark current RMS= 0.40 nA
Johnson-Nyquist RMS= 1.3 nA