# Solutions to selected problems in Photonics

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## 1 Exercise 17.1-4 Extraction efficiency (p. 690)

b) The critical angle when going from a material with refractive index $n$ to air (vacuum) is given by

$$\Theta_c = sin^{-1}(1/n) \tag{1}$$

The fraction of light that can be extracted is here denoted $\eta_3$ and is given by the solid angle of the light cone that does not undergo total internal reflection. It is given by

$$\eta_3 = \frac{1}{2}(1 - \sqrt{1 - \frac{1}{n^2}}) \tag{2}$$

The table below shows the critical angles and the extraction efficiencies for the various materials.

| material | $n$ | $\Theta_c$ | $\eta_3$ (%) |
|---|---|---|---|
| GaAs | 3.5 | 16 | 2 |
| GaN | 2.5 | 24 | 4 |
| Polymer | 1.5 | 42 | 13 |

c) The combined effect of refraction at the semiconductor-polymer interface and at the polymer-air interface give exactly the same escape cone as the bare semiconductor surface. The enhancement in using a polymer is then either a reduction in Fresnel reflection as discussed in (d) or in the possibility to shape the polymer to a non-planar surface as shown in Figure 17.1-12 (b).

## 2 Exercise 17.2-5 Amplifier gain and bandwidth (p. 746)

The net gain $\gamma_0(\nu)$ for a (bulk) GaAs amplifier is given by (eq. 17.2-4)

$$\gamma_0(\nu) = \frac{\lambda^2}{8\pi\tau_r}\varrho(\nu) f_g(\nu) \tag{3}$$

At $T = 0$K the Fermi inversion factor $f_g(\nu)$ becomes

$$f_g(\nu) = \begin{cases} 1 & h\nu < E_{fc} - E_{fv} \\ -1 & \text{otherwise} \end{cases} \tag{4}$$

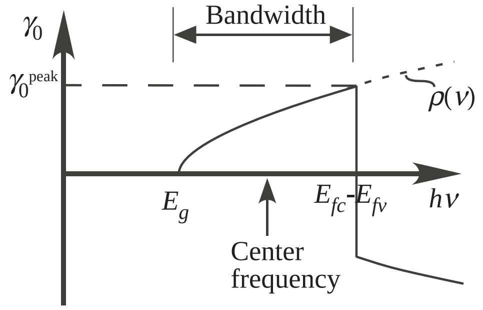


Figure 1: Gain spectrum

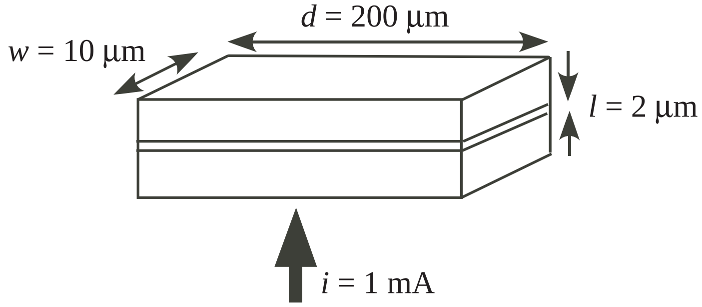


Figure 2: Schematic of the amplifier

We can then plot $\gamma_0(\nu)$ in Figure **??**.

A schematic description of the amplifier is given in Figure **??**.

This geometry gives a concentration of injected carriers

$$\Delta n = \frac{\tau i}{eldw} = \frac{50 \cdot 10^{-9} \cdot 10^{-3}}{1.6 \cdot 10^{-19} \cdot 2 \cdot 10 \cdot 200 \cdot 10^{-18}} \text{ m}^{-3} = 7.8 \cdot 10^{22} \text{ m}^{-3} \tag{5}$$

The separation of the quasi-Fermi levels is then (e. g. from exercise 17.1-1)

$$E_{fc} - E_{fv} = E_g + (3\pi^2)^{2/3} \frac{\hbar^2}{2m_r} \Delta n^{2/3} = E_g + 11 \text{ meV} \tag{6}$$

(Remember the reduced mass $m_r$ defined by $\frac{1}{m_r} = \frac{1}{m_c} + \frac{1}{m_v}$)

a) The bandwidth is $E_{fc} - E_{fv} - E_g = 11$ meV or, in frequency, $\Delta\nu = 2.7$ THz.

The center frequency is given by

$$\nu_c = \frac{E_g}{h} + \frac{\Delta\nu}{2} = 345 \text{ THz} \tag{7}$$

The peak net gain $\gamma_0^{\text{peak}}$ is

$$\gamma_0^{\text{peak}} = \gamma_0(E_{fc} - E_{fv}) = \frac{\lambda^2}{8\pi\tau_r} \frac{(2m_r)^{3/2}}{\pi\hbar^2} \sqrt{\underbrace{h\Delta\nu}_{11 \text{ meV}}} \tag{8}$$

Here $\lambda = \frac{hc}{E_g + 11 \text{ meV}}$ =867 nm and thus

$$\gamma_0^{\text{peak}} = \frac{(867 \cdot 10^{-9})^2}{8\pi \cdot 50 \cdot 10^{-9}} \frac{(2 \cdot 0.06 \cdot 9.1 \cdot 10^{-31})^{3/2}}{\pi (1.05 \cdot 10^{-34})^2} \sqrt{11 \cdot 10^{-3} \cdot 1.6 \cdot 10^{-19}} \text{ m}^{-1} = 260 \text{ cm}^{-1} \tag{9}$$

b) Number of voice messages: $\Delta\nu / 4$ kHz $= 7 \cdot 10^8$ (!)

c) Bit rate: 64 kbit/s$\cdot 7 \cdot 10^8 = 43$ Tbit/s

## 3 Exercise 17.3-2 Longitudinal modes (p. 747)

As in the previous exercise, the bandwidth is given by the difference between the quasi-Fermi level separation $E_{fc} - E_{fv}$ and the bandgap $E_g$

$$\Delta\nu = (E_{fc} - E_{fv} - E_g/h = (0.96 - 0.91) \text{ eV}/h = 12 \text{ THz} \tag{10}$$

The mode spacing is $\nu_F = \frac{c_0}{2nd} = \frac{3 \cdot 10^8}{2 \cdot 3.5 \cdot 250 \cdot 10^{-6}}$ s$^{-1}$ = 170 GHz. The maximum number of modes is therefore $\frac{12 \text{ THz}}{170 \text{ GHz}} = 70$.

## 4 Exercise 17.3-3 Minimum gain for lasing (p. 747)

The reflection loss coefficient is (eq. 17.3-3)

$$\alpha_m = \frac{1}{2d} \ln\left(\frac{1}{R_1 R_2}\right) \tag{11}$$

To get laser action, the gain coefficient has to be at least as large as the reflection loss coefficient.

The reflectances $R_1 = R_2$ are given by the difference in refractive index of the semiconductor and the surrounding air

$$R_1 = R_2 = \left(\frac{n-1}{n+1}\right)^2 = \left(\frac{3.5-1}{3.5+1}\right)^2 = 0.3 \tag{12}$$

Thus

$$\gamma_0^{\text{lasing}} > \alpha_m = \frac{1}{2 \cdot 500 \cdot 10^{-6}} \ln\left(\frac{1}{0.3^2}\right) \text{ m}^{-1} = 24 \text{ cm}^{-1} \tag{13}$$