

Solutions to selected problems in Photonics

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1 Exercise 16.1-4 EHP injection in GaAs (p. 648)

a) The law of mass action $n_0 p_0 = n_i^2$ gives, with $n_i = 1.8 \cdot 10^6 \text{ cm}^{-3}$ from Table 16.1-3,

$$p_0 = \frac{n_i^2}{n_0} = \frac{(1.8 \cdot 10^6)^2}{10^{16}} \text{ cm}^{-3} = 3.2 \cdot 10^{-4} \text{ cm}^{-3} \quad (1)$$

b) The recombination lifetime is given by

$$\tau = \frac{1}{r(n_0 + p_0 + \Delta n)} \quad (2)$$

With low-level injection, this expression is often dominated by the majority carriers. However, as will be clear in c), this is not the case here. The problem therefore becomes

$$\tau = \frac{1}{r(n_0 + p_0 + R\tau)} \Rightarrow \tau = -\frac{n_0}{2R} \pm \sqrt{\frac{1}{rR} + \frac{(n_0)^2}{4R^2}} \quad (3)$$

Putting in the numbers will give $\tau = 1 \mu\text{s}$.

c) The excess carrier concentration is $\Delta n = R \cdot \tau = 10^{17} \text{ cm}^{-3}$ which is larger than n_0 .

d) Using the expressions for the quasi-Fermi levels given in Exercise 16.1-3 (p. 645) we can write

$$E_{fc} - E_{fv} = E_c + (3\pi^2)^{2/3} \frac{\hbar^2}{2m_c} n^{2/3} - E_v + (3\pi^2)^{2/3} \frac{\hbar^2}{2m_v} p^{2/3} \quad (4)$$

$$= E_g + \left(\frac{1}{m_c} + \frac{1}{m_v}\right) (3\pi^2)^{2/3} \frac{\hbar^2}{2} \Delta n^{2/3} \quad (5)$$

where it was assumed that $\Delta n \gg n_0$ and p_0 . Inserting the numbers give $E_{fc} - E_{fv} = 1.433 \text{ eV}$, that is, only slightly more than the bandgap. Due to the difference in effective mass, E_{fc} will be further in the conduction band than E_{fv} is in the valence band.

2 Exercise 16.1-8 E-H recombination under strong injection (p. 678)

The equation that describes the time evolution of the excess charge carriers is

$$\frac{d\Delta n}{dt} = R - \frac{\Delta n}{\tau} \quad (6)$$

Here we are interested in the time immediately after turning of some high-level injection source. Thus $R = 0$ and $\tau = 1/r\Delta n$ and therefore

$$\frac{d\Delta n}{dt} = -r\Delta n^2 \quad (7)$$

The solution to this differential equation is of the form

$$\Delta n(t) = \frac{1}{A+Bt} \Rightarrow \frac{d\Delta n}{dt} = \frac{-B}{(A+Bt)^2} \quad (8)$$

Inserting these expressions in the differential equation Eq ?? yields $B = r$. With Δn_0 denoting the initial excess carrier concentration, i. e. $\Delta n_0 \equiv \Delta n(t_0)$ we can write

$$\Delta n(t) = \frac{\Delta n_0}{1 + r\Delta n_0 \cdot (t - t_0)} \quad (9)$$

Note that eventually the excess carrier concentration will become lower than the equilibrium majority carrier concentration so that τ will be a constant. This will change the differential equation and its solution will instead be a "normal" exponential decay.

3 Exercise 16.2-1 Photon emission rate > absorption rate (p. 667)

a) The difference between the occupation probabilities for emission, $f_e(\nu)$, and for absorption, $f_a(\nu)$, has to be positive for the emission rate to exceed the absorption rate:

$$f_e(\nu) - f_a(\nu) = f_c(E_2)[1 - f_v(E_1)] - [1 - f_c(E_2)]f_v(E_1) = f_c(E_2) - f_v(E_1) = f(E_2) - f(E_1) \quad (10)$$

Here we used the keyword "thermal equilibrium", i. e. that the same Fermi-function $f(E)$ (the same Fermi-level E_f) characterise both the valence band and the conduction band. Since the Fermi-function decreases monotonically with increasing energy the difference $f(E_2) - f(E_1)$ is always negative if E_2 is a conduction band state and E_1 a valence band state.

b) Forming the same difference as in a) we get:

$$f_e(\nu) - f_a(\nu) = f_c(E_2) - f_v(E_1) = \frac{1}{1 + e^{(E_2 - E_{fc})/kT}} - \frac{1}{1 + e^{(E_1 - E_{fv})/kT}} \quad (11)$$

This is positive if

$$e^{(E_2 - E_{fc})/kT} < e^{(E_1 - E_{fv})/kT} \Rightarrow E_2 - E_{fc} < E_1 - E_{fv} \Rightarrow h\nu = E_2 - E_1 < E_{fc} - E_{fv} \quad (12)$$

where we have used that $E_2 - E_1 = h\nu$. The result implies that at least one of the quasi-Fermi levels has to be in the corresponding band. Since the electron mass is normally smaller than the hole mass, the usual requirement to get an emission rate that exceeds the absorption rate is that $E_{fc} > E_c$.

4 Exercise 17.1-1 Quasi-Fermi levels of a pumped semiconductor (p. 685)

a) At $T=0K$ the Fermi-distribution reduces to a step function and the electron concentration can thus be obtained as

$$n = \int_{E_c}^{\infty} \rho_c(E) f_c(E) dE = \int_{E_c}^{E_{fc}} \frac{(2m_c)^{3/2}}{2\pi^2 \hbar^3} \sqrt{E - E_c} dE = \frac{(2m_c)^{3/2}}{3\pi^2 \hbar^3} (E_{fc} - E_c)^{3/2} \quad (13)$$

where $\rho_c(E)$ was replaced by the normal three-dimensional density of states. Since there are no thermally generated electrons, we can set $n = \Delta n$ in the expression above which after some manipulation gives

$$E_{fc} - E_c = (3\pi^2)^{2/3} \frac{\hbar^2}{2m_c} \Delta n^{2/3} \quad (14)$$

Similarly

$$p = \int_{-\infty}^{E_v} \rho_v(E) [1 - f_v(E)] dE = \int_{E_{fv}}^{E_v} \rho_v(E) dE = \frac{(2m_v)^{3/2}}{3\pi^2 \hbar^3} (E_v - E_{fv})^{3/2} \quad (15)$$

which, with $p = \Delta n$, gives

$$E_v - E_{fv} = (3\pi^2)^{2/3} \frac{\hbar^2}{2m_v} \Delta n^{2/3} \quad (16)$$

The difference in quasi-Fermi levels is thus

$$E_{fc} - E_{fv} = \underbrace{E_c - E_v}_{E_g} + (3\pi^2)^{2/3} \left(\frac{\hbar^2}{2m_c} + \frac{\hbar^2}{2m_v} \right) \Delta n^{2/3} = E_g + (3\pi^2)^{2/3} \frac{\hbar^2}{2m_r} \Delta n^{2/3} \quad (17)$$

where m_r is the reduced effective mass.

b) The occupation function relevant for emission is the product of the probability that the conduction band state with energy E_2 is occupied and the probability that the valence band state with energy E_1 is empty. At $T=0 K$ this simplifies to

$$f_e(\nu) = f_c(E_2) [1 - f_v(E_1)] = \begin{cases} 1 & E_2 < E_{fc} \text{ and } E_1 > E_{fv} \\ 0 & \text{otherwise} \end{cases} \quad (18)$$

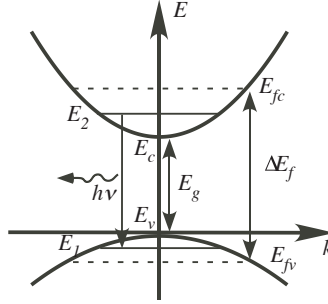


Figure 1: Band structure

How are these different energies related? Consider the following band structure

Here the effective masses (the "curvatures") determine how deep into the bands the Fermi-levels will be for a given concentration of injected charge

$$E_{fc} - E_c = \frac{m_r}{m_c} \underbrace{(E_{fc} - E_{fv} - E_g)}_{\Delta E_f} \quad (19)$$

However, the effective masses also determine how deep into the bands E2 and E1 will be, given a photon energy $h\nu$

$$E_2 - E_c = \frac{m_r}{m_c} (h\nu - E_g) \quad (20)$$

Thus

$$E_{fc} - E_2 = \frac{m_r}{m_c} (\Delta E_f - h\nu) > 0 \text{ if } h\nu < \Delta E_f \quad (21)$$

The same holds for E_{fv} and E_1

$$E_1 - E_{fv} = \frac{m_r}{m_v} (\Delta E_f - h\nu) > 0 \text{ if } h\nu < \Delta E_f \quad (22)$$

Therefore

$$f_e(\nu) = \begin{cases} 1 & h\nu < \Delta E_f \\ 0 & \text{otherwise} \end{cases} \quad (23)$$

We are now ready to plot Figure ???. These plots are similar to Figure 17.2-2. What are the differences?

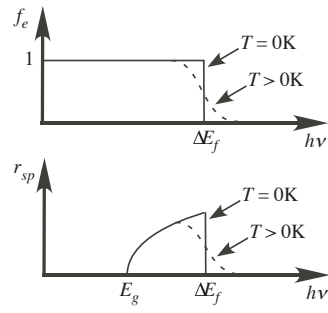


Figure 2: Occupation function and emission rate