Exercises week 5: Fiber communication

<u>Problem 1 (FoP 9.3-3):</u> **Scattering loss.** At λ_0=820 nm the absorption loss of a fiber is 0.25 dB/km and the scattering loss is 2.25 dB/km. If the fiber is used instead at λ_0=600 nm, and calorimetric measurements of the heat generated by light absorption give a loss of 2 dB/km, estimate the total attenuation at λ_0=600 nm.

<u>Problem 2 (FoP 23.2-1):</u> **Cascaded MZI MUX/DEMUX.** Three Mach-Zehnder interferometers (MZIs) are cascaded as shown in Fig. 23.2-6 to multiplex or demultiplex four wavelength channels with wavelength separation $\Delta\lambda$=0.2 nm and central wavelength 1550 nm. Determine the necessary path length differences Δd in each interferometer if the refractive index is n=2.3.

<u>Problem 3 (FoP 23.3-1):</u> **Power loss and crosstalk.** A 4x4 switch may be implemented by use of five 2x2 switches. If each of these switches introduces a power loss of 0.5 dB and a crosstalk of -30 dB, determine the worst case power loss and crosstalk for a 4x4 switch.

<u>Problem 4:</u> **Interconnects, routers, and switches.**

An integrated Mach-Zehnder interferometer is used to separate two wavelengths, λ_1 and λ_2, incident at one input port to either of the output ports (see fig. 1). λ_1=1550 nm, the path length difference is Δd=500 µm, and the refractive index is n=1.5. What must the wavelength λ_2 be in order to go to the other port? Are there several choices for λ_2? What must a third wavelength λ_3 be in order to go to the same port like λ_1? Is the total length d_0 of the device important for its function?

A nonlinear Mach-Zehnder interferometer is used for intensity-based routing (see fig. 2). Pulses with intensity I_1 go to output port 1 and pulses with intensity $I_2=I_1/2$ go to output 2. The nonlinear element, placed in one of the interferometer arms, has a nonlinear refractive index of $n_2=10^{-11}$ cm^2/W and is 1 mm long. At low intensity both interferometer arms have identical length. Find the intensities I_1 and I_2. Assume λ=1550 nm. (Remember that the refractive index changes as $n=n_0+n_2I$.)

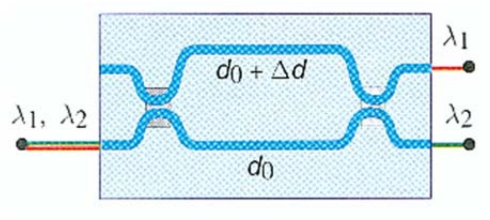

Fig 1: 2x2 optical coupler used as multiplexer.

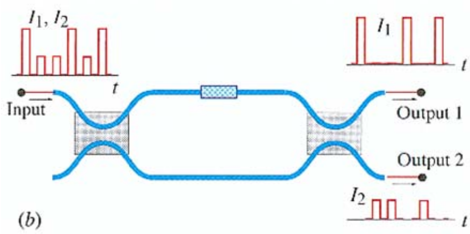

Fig. 2: Intensity-based routing

<u>Problem 5:</u> **Fibre optical communication systems.**

Power budget and time budget: We look at a fiber communication link operating at λ=1550 nm with a data rate of B_0=5 Gb/s. The link is spliced together from 10 km long pieces of fiber with losses of 0.2 dB/km and a dispersion coefficient D_λ=17 ps/km-nm. The coupling losses between the fiber pieces are 0.2 dB per connection, the in/out-coupling losses are 0.5 dB and we consider a power margin of 6 dB. The source is a distributed feedback laser with spectral width σ_λ=0.1 nm, a response time of 20 ps, and a power of 1 mW. The detector requires 1.3 µW of optical power at the output of the link and it has a response time of 100 ps.

Calculate the power and time budget and find out, if the system is attenuation or dispersion limited. Use the criterion that the response time of the overall system does not exceed 70% of the bit time. (Use $\sigma_s \sigma_\nu$=0.375 to find the Fourier limited pulse duration σ_s of the source.)

For the above link, if we replace the fiber by a dispersion shifted fiber with D_λ=1 ps/km-nm, what is the maximum dispersion limited length of the link? The link now obviously is attenuation limited. How many 20 dB gain Erbium amplifier segments would one need to splice into the link in order to remove the attenuation limitation (Let's forget about splicing losses and noise from the amplifier here!)?

Short answers:

Problem 1 (FoP 9.3-3): 9.85 dB/km at 600 nm.

Problem 2 (FoP 23.2-1): $\Delta d = 2.6$ mm for the first interferometer and $\Delta d = 1.3$ mm for the second interferometer.

Problem 3 (FoP 23.3-1): maximum loss of 1.5 dB. Maximum crosstalk - 25.2 dB.

Problem 4: $\Delta\lambda = 1.6$ nm (at vacuum). $\lambda_2 = \lambda_1 \pm m\Delta\lambda$ with $m = 1,3,...$, e.g. λ_2=1551.6, 1548.4, 1554.8, 1545.2 nm. $\lambda_3 = \lambda_1 \pm 2m\Delta\lambda$ with $m = 1,2,....$, e.g. 1553.2, 1546.8 nm. The total length d_0 of the device is not important for its functioning (as long as it is shorter than the coherence length of the source).

$I_1 = \frac{\lambda}{n_2 d} \approx 1.55 \times 10^8$ Wcm^{-2}.

Problem 5: Power budget: $L \approx 100.3$ km, time budget: $L \approx 53.6$ km. The system is dispersion limited.

With the dispersion-shifted fiber: L=911 km, 9 EDFA amplifiers are needed along the way.

Detailed solutions

Problem 1 (FoP 9.3-3): We know that the scattering loss scales as $1/\lambda^4$. Knowing that the scattering loss at 820 nm is 2.25 dB/km, we obtain 7.85 dB/km at 600 nm, resulting in a total attenuation of 9.85 dB/km at 600 nm.

Problem 2 (FoP 23.2-1): The first interferometer should separate wavelength channels with a spacing of $\Delta\lambda$=0.2 nm from each other (i.e. λ_1 from λ_2 and λ_3 from λ_4). Note that the channel spacing $\Delta\lambda=\Delta\lambda_0$ normally refers to the vacuum wavelength, whereas the wavelength in the material is different. The frequency difference is $\Delta\nu = c_0/2n\Delta d$ (FoP 23.2-1). One can use $\Delta\nu/\nu \approx \Delta\lambda_0/\lambda_0 \Leftrightarrow \Delta\nu = c_0\,\Delta\lambda_0/\lambda_0^2$ to obtain $\Delta d = \lambda_0^2/2n\Delta\lambda_0 = 2.6$ mm. The second interferometer separates channels with spacing $2\Delta\lambda_0$=0.4 nm. The physical length difference is $\Delta d = 1.3$mm.

Problem 3 (FoP 23.3-1): In the worst case the signal has to pass three 2x2 switches, resulting in a maximum loss of 1.5 dB. The maximum crosstalk is 3×10^{-3}, corresponding to $10\log(3\times10^{-3}) = -25.2$ dB.

Problem 4: Like in problem 2 we use $\Delta\nu = c_0/2n\Delta d = 200$ GHz. With $\Delta\nu/\nu \approx \Delta\lambda_0/\lambda_0 \Leftrightarrow \Delta\nu = c_0\,\Delta\lambda_0/\lambda_0^2$ we obtain $\Delta\lambda = 1.6$ nm (at vacuum). Possible choices for the wavelength that goes to the other output port are $\lambda_2 = \lambda_1 \pm m\Delta\lambda$ with $m = 1,3,\ldots$, e.g. λ_2=1551.6, 1548.4, 1554.8, 1545.2 nm. Wavelengths that go to the same port like λ_1 can be $\lambda_3 = \lambda_1 \pm 2m\Delta\lambda$ with $m = 1,2,\ldots$., e.g. 1553.2, 1546.8 nm. The total length d_0 of the device is not important for its functioning, if the source is sufficiently coherent.

We are looking for the intensities I_1 and I_2 that lead to an increase of the path length of the interferometer arm with the nonlinear element by a full wavelength and half a wavelength, respectively. We can write for the nonlinear phase shifts $\Delta\varphi_1 = \frac{2\pi}{\lambda_0} n_2 I_1 d$ and $\Delta\varphi_2 = \frac{2\pi}{\lambda_0} n_2 I_2 d$. The difference between the two phase shifts must be π:

$$\Rightarrow \Delta\varphi_1 - \Delta\varphi_1 = \pi \Leftrightarrow \frac{2\pi}{\lambda} n_2 d\left(I_1 - \frac{I_1}{2}\right) = \pi \Leftrightarrow I_1 = \frac{\lambda}{n_2 d} \approx 1.55\times10^8 \text{ Wcm}^{-2}.$$

Problem 5: We start by writing the source and detection powers in dB using the definition on page 1091 (FoP). $\Rightarrow P_s = 0$ dB and $P_d = -28.86$ dB. Note that the definition how optical power is translated to dB is totally arbitrary. We can now write down the power budget:

$$P_s - 2P_{io} - L\alpha - \left(\frac{L}{10} - 1\right)P_c - P_{saf} = P_d$$

$$\Leftrightarrow L = \frac{P_s - 2P_{io} - P_{saf} - P_d + P_c}{\alpha + P_c/10} \approx 100.3km$$

The coupling losses between the 10 km long fiber pieces was taken into account in a rudimentary manner. The particular way of how this is done might change the result slightly. In order to write down the time budget, we need the duration of the source σ_s, which can be obtained from the time-bandwidth product $\sigma_s \sigma_\nu = 0.375$ with $\frac{\Delta\nu}{\nu} \approx \frac{\Delta\lambda}{\lambda}$ we obtain $\sigma_\nu = c_0 \frac{\sigma_\lambda}{\lambda_0^2} = 12.5$ GHz and the transform-limited duration of the source is $\sigma_s = 30$ ps. We can now write down the time budget:

$$\sigma_s^2 + \sigma_{tx}^2 + \sigma_\tau^2 + \sigma_{rx}^2 = (0.7T)^2 = \left(\frac{0.7}{B_0}\right)^2$$

$$\Leftrightarrow (LD_\lambda \sigma_\lambda)^2 = \left(\frac{0.7}{B_0}\right)^2 - \sigma_s^2 - \sigma_{tx}^2 - \sigma_{rx}^2$$

$$\Leftrightarrow L = \frac{\sqrt{\left(\frac{0.7}{B_0}\right)^2 - \sigma_s^2 - \sigma_{tx}^2 - \sigma_{rx}^2}}{D_\lambda \sigma_\lambda}$$

$$\Leftrightarrow L \approx 53.6\ km$$

The system is dispersion limited.

If we use a dispersion-shifted fiber with D_λ=1 ps/km-nm instead, the dispersion limited length suddenly becomes L=911 km. Now the system obviously is attenuation limited. If we look at the power budget with the new length, we recognize that an amplification of 178.3 dB is needed for the signal to travel the new length. Thus, 9 EDFA amplifiers with 20 dB amplification each need to be distributed along the way.