Conduction in Quantum Point Contacts

14th December 2015

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FFF042
Physics of Low-Dimensional Structures and Quantum Devices

1 Explanation of System

A point contact in the macroscopic world can be thought of as a narrow conducting channel between two bulk metallic conductors, constricting the flow of electrons between them. A construction, similar in principle, can be made on a small scale in a two-dimensional electron gas (2DEG), with an electrostatically defined constriction. The 2D electron gas is created at the interface between e.g. AlGaAs and GaAs, and by adding a split gate on top of the GaAs–AlGaAs heterostructure, one may create a short and narrow constriction of the passage of electrons from a source to a drain. [5]

The point contact is considered to be connected to two large reservoirs, a source and a drain, characterized by their temperatures T_S and T_D, and their chemical potentials μ_S and μ_D, through which the distribution of the electrons in the source and the drain is given as the Fermi distribution function. The reservoirs act as either sources of electrons, according to the Fermi distribution, or as sinks, catching electrons. [1]

The constrictions in the considerations presented hereafter are induced electrostatically and therefore the potential is a smooth function without any hard walls or corners. The split gate creates a smooth bulge in the longitudinal direction and, in the narrow bottleneck, a smooth well in the transverse direction. Thus, the potential faced by incoming electrons is shaped as a saddle, given near the bottleneck as

$$V(x,y) = V_0 - \frac{1}{2}m\omega_x^2 x^2 + \frac{1}{2}m\omega_y^2 y^2, \tag{1}$$

where x has been defined as the longitudinal direction and y as the transverse direction, V_0 is the potential at the saddle, and ω_x and ω_y define the curvature of the saddle. The shape of the potential can be seen in Fig. 1. [2]

The total energy can be obtained by adding a kinetic energy $p^2/2m$ to the potential in Eq. (1). One may then separate the Hamiltonian into a transverse and a longitudinal wave function.[2] In the transverse direction, there is a parabolic well, solutions to which can be obtained as for the quantum harmonic oscillator, rendering energies $\epsilon_n = \hbar\omega_y (n + 1/2)$ for $n = 0, 1, 2, \ldots$ [3].

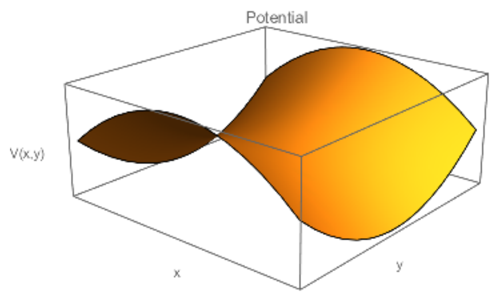


Figure 1: *3D plot of the saddle potential. Note that x is the longitudinal direction.*

In the longitudinal direction, an electron moves in the effective potential $V_0 + \hbar\omega_y(n + \frac{1}{2}) - \frac{1}{2}m\omega_y^2 x^2$, which may be regarded as the band bottom of a quantum channel around the saddle point. Disregarding tunnelling, electrons may be said to be able to pass the through the channel if they have an energy larger than the threshold energy

$$E_n = V_0 + \hbar\omega_y \left(n + \frac{1}{2}\right), \tag{2}$$

and unable to do so if their energy is below the same. Thus, in a system, channels with threshold energies below the Fermi energy can be considered open, whilst channels with threshold energies above the same are closed. [2]

2 Theoretical Background

2.1 Conduction

Naturally, the picture may be complicated further. Consider quantum mechanical transmission and reflexion of the wave functions of incoming electrons. This allows for channels that are neither fully open nor fully closed, but that allow transmission between channel m and n with a probability T_{mn}.

One may derive the quantum statistical average of the current going into the sample, and Blanter et al. does in Ref. [1], as

$$\langle I_L \rangle = \frac{e}{2\pi\hbar} \int dE \, \mathrm{Tr}[t^\dagger(E)t(E)][f_S(E) - f_D(E)], \tag{3}$$

where t is the transmission matrix, and $t^\dagger$ its Hermitian adjoint. If one then, for the sake of simplicity, considers the zero-temperature limit, the Fermi distributions becomes Heaviside functions, and if one furthermore considers very small voltages V between the source and the drain, Eq. (3) is simplified considerably. Thereby, one may define conductance as

$$G = \frac{e^2}{2\pi\hbar} \mathrm{Tr}[t^\dagger(E_F)t(E_F)]. \tag{4}$$

By diagonalizing the matrix $t^\dagger t$—which may be done as any matrix multiplied by its Hermitian adjoint renders a Hermitian matrix, which may be diagonalized—one obtains a set of eigenvalues T_n that are the transmission probabilities. This simplifies the expression for the conductance further to

$$G = \frac{e^2}{2\pi\hbar} \sum_n T_n. \tag{5}$$

[1]

The potential is quadratic, and analysis thereof would show that there is no channel mixing, i.e. the transmission probability is non-zero only if the incoming and outgoing channels are the same. The transmission probabilities may be expressed as

$$\varepsilon_n = \frac{2}{\hbar\omega_x} \left[E - V_0 - \hbar\omega_y \left(n + \frac{1}{2} \right) \right], \tag{6}$$

$$T_n = \frac{1}{1 + \exp(-\pi\varepsilon_n)}. \tag{7}$$

If plotted, T_n looks like a step, similar to the Fermi distribution function. Summing T_n over n yields a sum of translated steps. Thus, by Eq. (5), the conductance is quantized. [2] The soft transitions of the steps come from tunnelling, and is therefore dependent on the shape of the barrier in the longitudinal direction; if the barrier is wide, electrons have a small probability of tunnelling through the barrier, and vice versa. [3]

2.2 Shot Noise

Closer inspection of the observed current will reveal that the previously derived laws are not in complete agreement with reality. This is because the current previously derived is the average current, not the absolute value of the current as a function of time. This is not possible to derive due to the quantum mechanical nature of the system; the state describing current flow is not a single quantity, but a superposition of various possible outcomes for the action of an electron upon crossing the barrier. However, one can derive the statistical deviation from the average value, by considering the mean squared error in the calculation.

We start with the mean squared error in the occupation number of the transmitted (n_T) and reflected (n_R) states, and use the fictitious but illustrative thought experiment in which a single electron is incident upon the barrier, and is either reflected or transmitted, with probabilities T and R respectively. Since there is only a single electron, the occupation number is either zero, or one, meaning the average of the occupation number is the same as the average of the square of the occupation number, since, obviously $1^2 = 1$ and $0^2 = 0$. Using this information we can then derive the the expression:

$$\langle (\Delta n_T)^2 \rangle = T(T - 1), \tag{8}$$

where $\Delta n_T = n_T - \langle n_T \rangle$. Hopefully this is in some way instructive, as to understanding why, when all the proper quantum mechanical considerations are made [1], the shot noise at low temperature for a two-terminal system is given by:

$$S = \frac{e^3|V|}{\pi\hbar} \sum_n T_n(1 - T_n) \tag{9}$$

The coefficients are a result of the derivations [1] and the summation comes from the same diagonalization of the relevant matrix, as well performed in the previous section. We can see here in Eq. (9) that the same form occurs, as when considering the mean squared error of the occupation number. This is to be expected as the shot noise is essentially the deviation of the current from the average current, which can in some way be expressed by the occupation number of transmitted electrons.

3 Visualization

In this section, we shall attempt to instill a greater understanding in the reader of how the various parameters affect the current, and the physical reasoning behind this. The most basic and defining feature of the system is the saddle potential created by the split gate. Such a potential is described by Eq. (1) earlier in this report and can be easily graphed in 3D, as shown in figure 1. This potential has two independent parameters which control its shape, ω_x and ω_y. These parameters are the frequencies of two orthogonal and inversely related harmonic oscillators.

Fig. 2 shows a plot of conductivity as a function of $E - V_0$, where these parameters are the same as they are in Eq. (6). The reason for this is that the energy available for the electron to pass through the barrier is equivalent to this expression and is the main determining factor. Also, the conductance is expressed as a factor of fundamental constants, as these simply scale the values, and do not affect the shape of the plot.

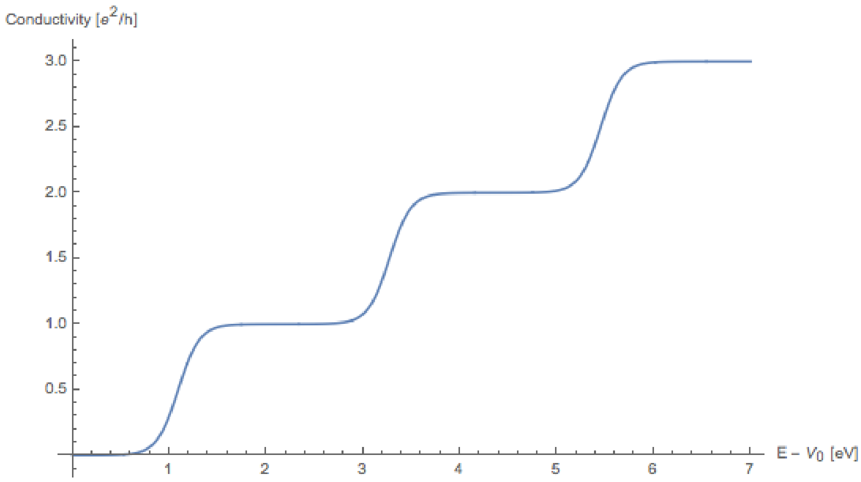


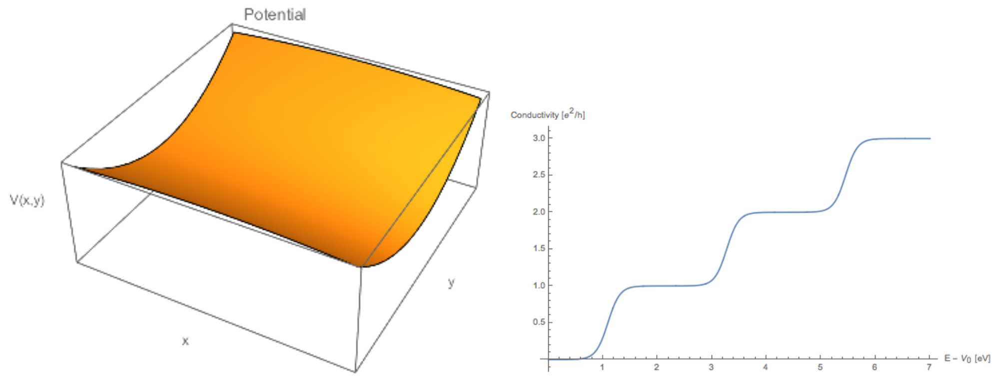
Figure 2: *Conductivity as a function of the electron's available energy. In this case the ratio* $\omega_y/\omega_x = 10$

Notice how the conductance is quantized[1] in units of e^2/h, this is a very significant feature of quantum point contacts, and this feature will be addressed again later in this report.

We will now investigate the effects of changing the ω_x and ω_y values, which will affect both the potential and the conductance.

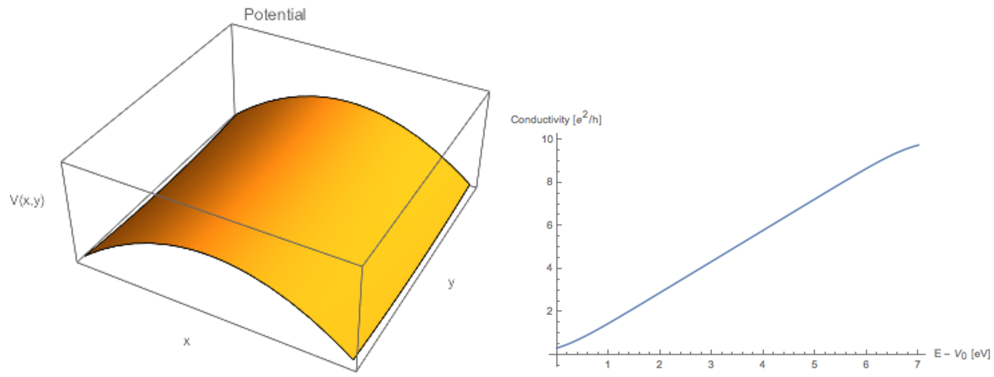
We can see clearly in Figs 3 and 4, that for a highly step like conductance, the potential is shaped much like a harmonic oscillator, and for the more linear shape of conductance, the potential is more like a hill. This implies that for a mix of the two structures, we have two concurrent phenomena. One which is the transmission of electrons through the waveguide like structure exhibited in Fig. 3a, which

[1] except at transition points



(a) *Saddle potential* **(b)** *Conductance*

Figure 3: *The relation between the shape of the potential and the conductance for a ratio* $\omega_y/\omega_x = 10$



(a) *Saddle potential* **(b)** *Conductance*

Figure 4: *The relation between the shape of the potential and the conductance for a ratio* $\omega_y/\omega_x = 0.1$

undergoes transmission for each conduction band with energy lower than the incoming electron, as shown in Fig. 5.

A low ω_y/ω_x ratio leads to the structure seen in Fig. 4a, through which quantum tunneling can occur. This leads to the more linear conductance plot, as tunneling is not a quantized phenomenon, it rises linearly as a function of the incoming electron energy. In the overall picture, these two separate phenomena combine to form a mixture of quantised and linear conductance.

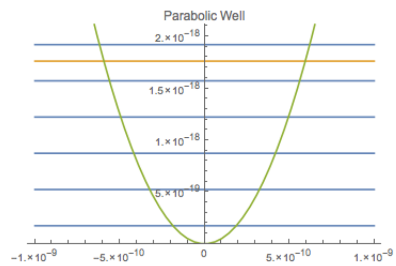


Figure 5: *Cross section of potential in x direction at y=0 (i.e.* ω_y *controlled part) with yellow line being* $E - V_0$*, and the blue lines being the quantised energies of the quantum harmonic oscillator*

We can now explore the relation between shot noise, and conductance, which is helpfully demonstrated by the use of the superimposed plot of the two, shown in Fig. 6. As can be seen in Fig. 6, the energies at which the shot noise is at a maximum is equivalent to the turning point in the conductance. One of the reasons for this is that these turning points are associated with the energy levels in the harmonic oscillator. As the small bias voltage passes over these lines, the current will become more random as the distribution in the energies of the incoming electrons will be spread over the energy level of the potential, as opposed to where the bias is completely above the level, meaning that all electrons can pass over the current energy level.

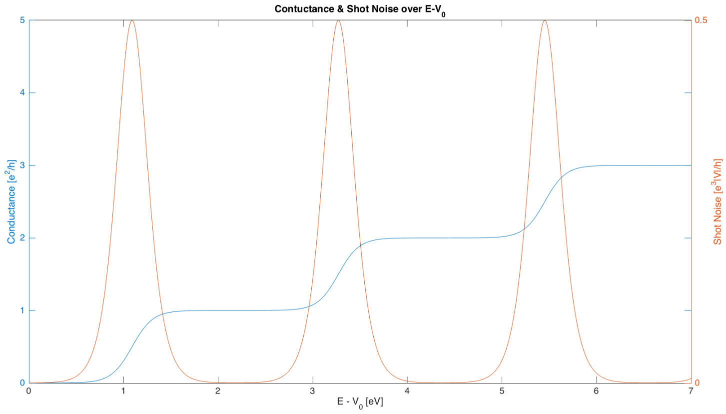


Figure 6: *Conductance and Shot noise, superimposed as a function of $E - V_0$. Note that these calculations are only valid for small $|V|$ and in this situation it is simply a scaling factor.*

4 Conclusion

This type of system is interesting for many different reasons. Firstly, the quantum mechanical nature of the device means that if small voltages are applied, a quantization of the available conductance can be observed spaced by e^2/h. One of the most interesting things about this phenomena is that this spacing is independent of any of the parameters affected by the material. It is a function only of fundamental constants. However, the energies at which these values for conductivity will be reached is affected by the material, as the material will affect the overall size of the potential given a specific gate voltage. There is also a gradual change between these levels, which is affected by the material as well.

QPC's have also been suggested as potential way of reading out information from quantum dot circuits, making them potentially useful in the field of Quantum Computation. They are useful in this application as when their conductance is plateaued they become extremely sensitive to changes in the electrostatic environment. The QPC is setup in such a way that if the charge of the quantum dot changes, the gate voltage (related directly to V_0) also changes, and hence the charge of the quantum dot can be discerned. Their main advantage over other methods of detecting the charge of quantum dots is that the charge of the dot can be detected without passing a current through it. This allows for readouts from the quantum dots when the leads controlling them are not interacting with the dot. [4]

References

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