Magneto-Characterization of a 2DEG

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***Abstract*—This report characterizes the electrical transport in a two dimensional electron gas (2DEG) using Shubnikov-de Haas (S-dH) and Hall effect measurements. The measurements have been done in a low-T fridge with a high perpendicular magnetic field. From the measurements, carrier density, mobility, Fermi wavelength and mean free path has been calculated and spin-separation at high magnetic fields has been observed.**

I. Introduction

THE characterization of low-dimensional structures is important since before a substrate can be used, its electrical transport characteristics have to be verified in order to get reliable data.

This report will describe the theory behind the types of measurements made, how they were made and what was measured. Afterwards, the results will be analysed and compared to the theory. Finally, the reason behind the differences between the results and theory will be hypothesized and a final analysis of the project as a whole will be made.

II. Theory

A. Two Dimensional Electron Gas

The sample used for the measurements is a semiconductor with different layers, see Fig. 1. In order for an electron gas to be considered 2D, it has to be confined in a space that is smaller than the mean free path of the electrons in one dimension. This is achieved by alternating GaAs and AlGaAs layers, creating a triangular quantum well at the interface. In order to attract electrons to the well, the AlGaAs is doped by Si. Even at low-T, some of the donors will be ionized, creating positive trapped ions that will attract electrons from the GaAs substrate to the well. The doping level of the Si is very sensitive, as the dopants will lower the conduction band of the AlGaAs and can therefore cause parallel conduction in the AlGaAs.

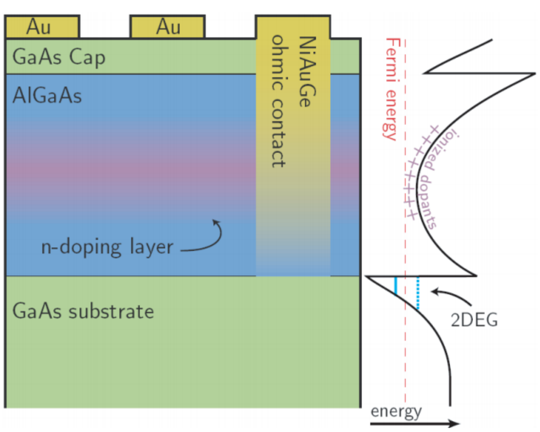


Fig. 1: Substrate used during measurements. The 2DEG is measured using the NiAuGe contact. Image from [1].

B. Landau Levels

When the Schrödinger equation is solved in a magnetic field, the continuous density of states (DOS) is replaced by δ-functions. These δ-peaks are called Landau levels (LL), see Fig. 2, and each contains a large number of degenerate states. These levels will have a non-zero width Γ due to electrons having a finite quantum lifetime τ_i given by (1)

$$\Gamma = \frac{\hbar}{\tau_i} \tag{1}$$

where $\hbar$ is Planck's reduced constant. The DOS n_B in a LL is given by (2)

$$n_B = \frac{eB}{h} = \frac{m}{\pi\hbar^2}\hbar\omega_c \quad \text{per spin} \tag{2}$$

where e is the elementary charge, B is the magnetic field, h is Planck's constant, m is the mass and ω_c is the cyclotron angular frequency. $\hbar\omega_c$ is the separation of the LLs and as long as $\hbar\omega_c < \Gamma$, n_B will be relatively constant. However as B is increased, the separation of the LLs as well as their heights will increase. As the density of electrons, n_{2D}, will not change; this will mean that not all LL states will be occupied as B is increased, given by (3)

$$\nu = \frac{n_{2D}}{n_B} = 2\pi l_B^2 n_{2D} \tag{3}$$

where ν is the filling factor (the number of occupied Landau levels) and l_B is the magnetic length.

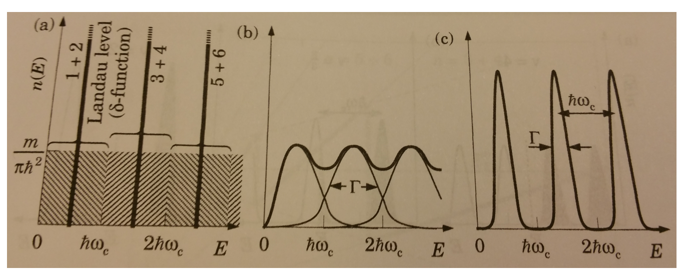


Fig. 2: Landau levels of different Γ. Image from [2].

There will be values of B where ν is an integer given by (4)

$$B_n = \frac{hn_{2D}}{en}, \quad n \in \mathbb{N} \tag{4}$$

where B_n is B at these values and n is the amount of full LLs. The LLs will create a few interesting effects. [2]

C. Shubnikov-de Haas Effect

By measuring longitudinal resistance R_{xx} in a 2D system with an applied, varied magnetic field, the Shubnikov-de Haas effect can be seen. As B is changed, it will reach B_n where $n_B = 0$. The proportionality of R_{xx} is given by (5)

$$R_{xx} \propto \rho_L \propto \sigma_L \propto n_B \tag{5}$$

where ρ_L is the longitudinal resistivity and σ_L is the longitudinal conductivity. This means that when $B = B_n$, $R_{xx} = 0$ will also apply. The resulting oscillation with zeroes of R_{xx} as a function of B is called the Shubnikov-de Haas effect. Minima of such oscillations occur at the fields B_n when $\nu = n$. This happens because longitudinal conduction occurs at the Fermi level and therefore disappears when the density goes to zero at $B = B_n$. Under these conditions, $\rho_L \propto \sigma_L$ and therefore vanishes too. Thus, a plot n against the measured values of B_n gives a straight line with the slope $(\pi\hbar/e) \cdot n_{2D}$ passing through the origin. This is the common way of measuring the density and also provides a clear demonstration that a system is two-dimensional [2]. So, in this case:

$$n_{2D} = \frac{e}{\pi\hbar \cdot \frac{(1/B)}{n}} \tag{6}$$

where ratio $\frac{(1/B)}{n}$ is the ratio of the slope in fan diagram mentioned above.

Another important parameter that characterizes 2DEG in given semiconductor structure is the mobility of charge-carriers. The effective mobility is given by (7) [2]

$$\mu = \frac{1}{n_{2D} e \rho_L} \tag{7}$$

where $\rho_L = r_0/d$. In this case, $d = L/W(L$ - length between two contacts in the Hall bar, W- cross-sectional area between two sides of the bar). r_0 is resistance without the presence of magnetic field, which can be obtained from the data.

Fermi wavelength is usually used to describe a wavelength of electrons that carry the current. Knowing the connected parameter Fermi wavenumber one can also determine the Fermi energy, velocity, mean free path and other parameters. Wavenumber can be deduced in the following way: It is known that the number of electrons N in quantum wells in 2D reciprocal space with wave vectors k_x and k_y is proportional to:

$$\frac{N}{2} = \pi k_F^2 \cdot \frac{L^2}{4\pi^2} \tag{8}$$

Factor 2 is because the states are doubly degenerate, $k_F^2 = k_x^2 + k_y^2$ is the radius of the circle and second factor resembles spacing between electrons in this reciprocal space. We can spot that the ratio N/L^2 is nothing more than electron density n_{2D}.

Finally, Fermi wavenumber can be calculated with (9)

$$k_F = \sqrt{2\pi n_{2D}}. \tag{9}$$

From wavenumber, it is not hard to get to the wavelength from (10)

$$\lambda = \frac{2\pi}{k_F}. \tag{10}$$

Mean free path is also an important characteristic that describes the distance that electrons in material can travel before scattering effects occur. It is usually given by (11)

$$l_{mfp} = v_F \cdot \tau \tag{11}$$

where v_F is the Fermi velocity, τ is relaxation time.

Relaxation time could be determined from the known formula for mobility:

$$\mu = \frac{e\tau}{m} \tag{12}$$

Fermi velocity:

$$v_F = \frac{\hbar k_F}{m} \tag{13}$$

In both equations $m = m^* m_0$, where m^* is the effective mass. Plugging equations (12) and (13) into (11) gives final expression for mean free path (14)

$$l_{mfp} = k_F \cdot \frac{\hbar\mu}{e}. \tag{14}$$

D. *Quantum Hall Effect*

By measuring the transverse resistance R_{xy} instead, the quantum Hall effect can be seen. R_{xy} is independent on geometry and is given by (15)

$$R_{xy} = \rho_T = \frac{B}{e n_{2D}} \tag{15}$$

where ρ_T is the transverse resistivity. This means that when $B = B_n$, (16) applies and R_{xy} will be quantized.

$$R_{xy}^{-1} = \left(\frac{e^2}{h}\right) n \tag{16}$$

The quantum Hall effect is more complex than that, though. There will be states between LLs, meaning that the plateaus should not be as well defined or long as they are. However these are edge states which can only propagate electrons in one direction. Because of the way the Hall bar in Fig. 3 in built, there will therefore be the same voltage across the entire top part of the bar decided by the voltage in the input stage and in the same way the entire bottom will have the same voltage as the output stage. This means that while the only available states are edge states, the longitudinal resistance is 0 and the transverse resistance is constant since there is no voltage difference along the longitude. This is called the quantum Hall effect. [2]

Knowing the fact that the charge carrier density remains constant, it is easy to determine it using non-quantized part of the hall resistivity versus magnetic field plot and do a regular Hall effect measurement. From (15), the density of charge carriers equals to:

$$n_{2D} = \frac{B}{e R_{xy}} \tag{17}$$

By plotting the R_{xy} vs B diagram, one could obtain the ratio between resistance and magnetic field, which is represented by the slope in the linear part of the plot.

E. *High-Field Effects*

In high B fields, the Landau peaks will begin to split up due to spin. This will cause splitting in R_{xx}, as will be shown later in Fig. 5 and 8a in Section IV. As the field grows, splitting becomes more apparent.

III. Experimental Method & Setup

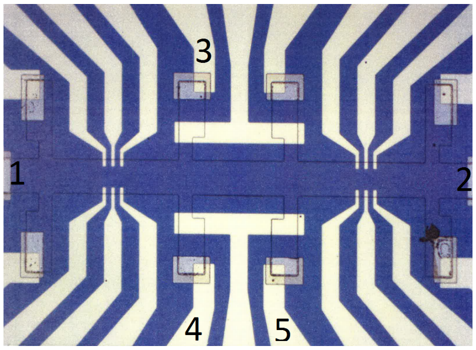


Fig. 3: The Hall bar sample used for the measurements.

The sample that was used in experiments is showed on Fig. 3. The alternating current flows from contact 1 to 2. The contacts used in measurements are labeled 3, 4 and 5, with 4 and 5 measuring Shubnikov-de Haas voltage, 3 and 4 measuring Hall voltage. To see quantum-mechanical behaviour of electrons in magnetic field, the sample was kept in the fridge "Triton 200" under the temperature of 13 mK.

Further, the circuit details used in measurements are described. A current source was provided by 4 V oscillation from $SRS830$ lock-in dropped across 1:100 voltage divider and then forced through 10 $M\Omega$ resistor to get the value of 4 nA. Current out of the Hall bar was monitored using a $EG\&G$ 5209 lock-in amplifier and had a steady value of 4 nA throughout the measurements. Both Hall and Shubnikov-de Haas voltages were on their own $SRS830$ lock-in amplifier.

IV. Results & Discussion

A. Charge Carrier Density of 2DEG

With measurements done, the Hall and Shubnikov-de Haas resistances were plotted against magnetic field. Fig. 4 represents the low-field measurements, while Fig. 5 covers the high-field regime. By using techniques described in Section II, charge carrier densities were calculated with Shubnikov-de Haas measurements as well as Hall's.

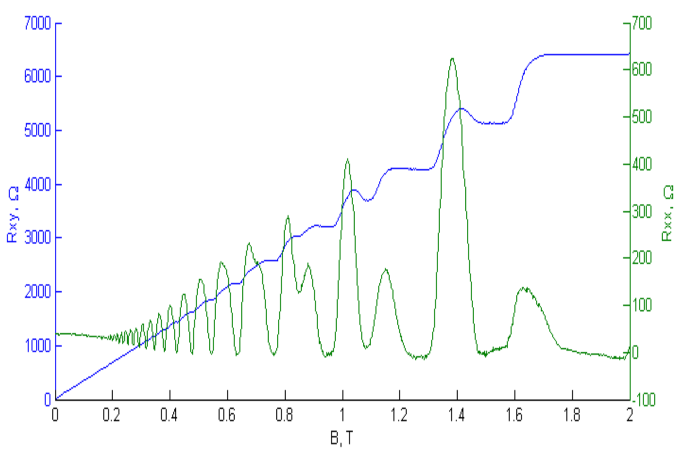


Fig. 4: Low-Field Hall and S-dH Resistance.

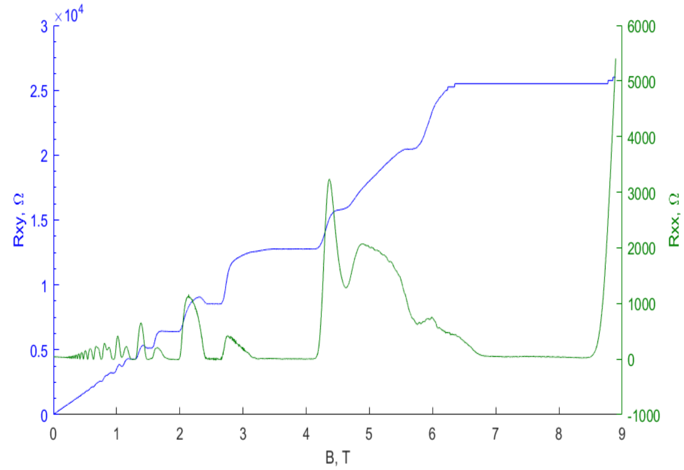


Fig. 5: High-Field Hall and S-dH Resistance.

1) Hall effect data analysis: With Hall effect resistance being linear in low-field range, we can do basic Hall measurements to determine charge-carrier density of our 2DEG. Measured values for output Hall voltage were divided by the value of current equal to 4 nA which ran through the bar. The resulting resitance values were plotted against magnetic field. The slope of the linear part represents the ratio R/B as it is shown on Fig. 6.

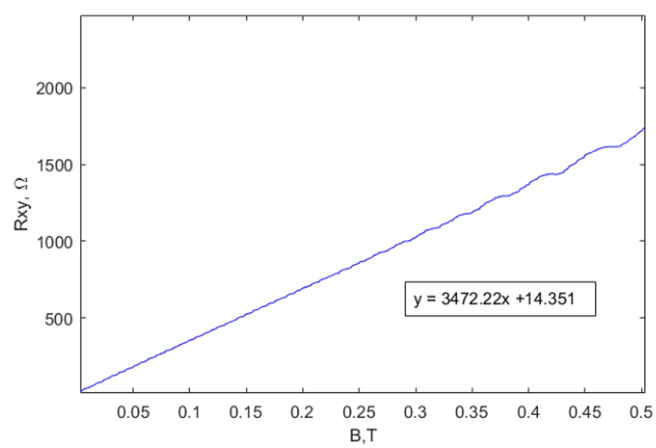


Fig. 6: R_{xy} vs. B diagram for Quantum Hall effect.

Thus, by using equation (17), the value for electron density was obtained: $n_H = 1.80 \cdot 10^{11}\ cm^{-2}$.

2) Shubnikov-de Haas data analysis: To get to the value of charge density using Shubnikov-de Haas effect data, usually more steps are required. Like in Hall's, Shubnikov-de Haas resistance was plotted (See green curve in Fig. 4 and Fig. 5.) To get a clear view of minima of Shubnikov-de Haas resistance oscillations, it is often convenient to plot R as a function of $1/B$ (Fig. 7).

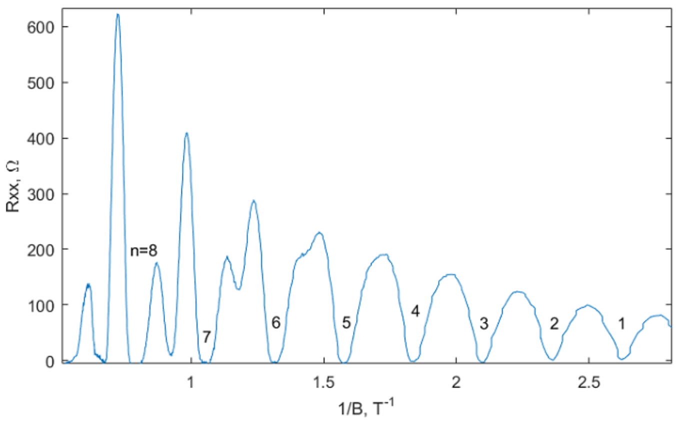


Fig. 7: R_{xx} vs. $\frac{1}{B}$ diagram for Shubnikov-de Haas effect.

Note that the first few minima in such plot could be deceiving due to spin splitting nature of Landau levels in higher fields. By labeling few minima and plotting them against corresponding values of magnetic field, one could get a straight line with slope $\frac{(1/B)}{n}$, as was discussed in Section II.

Thus, by applying the formula (6), value for density can be calculated: $n_S = 1.81 \cdot 10^{11}\ cm^{-2}$. Both measurements are in good connection with theoretical data (grower‘s expected value = $1.8 \cdot 10^{11}\ cm^{-2}$).

One can also note that the densities are almost the same for Hall and Shubnikov-de Haas, which also approves the fact that measurements were done correctly, density of electrons in 2DEG should be a constant value.

B. *Calculating mobility, mean free path and Fermi wavelength*

We will use Shubnikov-de Haas measurements to calculate the value for the mobility of charge carriers in this 2DEG. Knowing the electron density, by using (7) it is not hard to obtain $\mu = 2.6905 \cdot 10^6\ \frac{cm^2}{V \cdot s}$. Expected value is equal to $2.6860 \cdot 10^6\ \frac{cm^2}{V \cdot s}$, which confirms the validity of the method described in Section II.

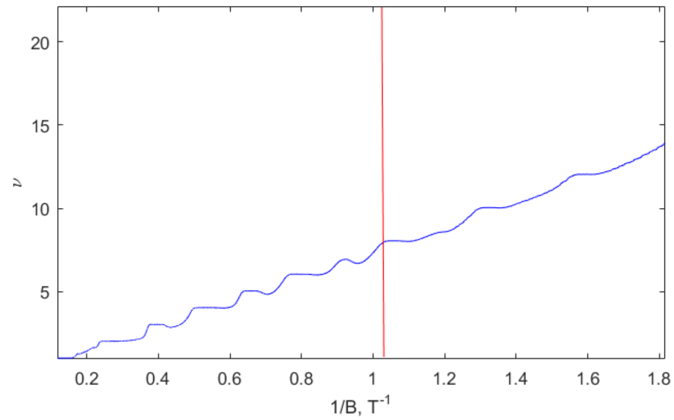
By using the obtained value for the charge carrier density, the Fermi wavenumber and thus Fermi wavelength can be calculated from (9) and (10) to $k_F = 1.06 \cdot 10^8\ m^{-1}$.

$\lambda_F = 5.92 \cdot 10^{-8}\ m = 59.2\ nm$, which is a reasonable value, considering the fact that in semiconductor structures, Fermi wavelengths lie within the range 10 - 100 nanometers [3].

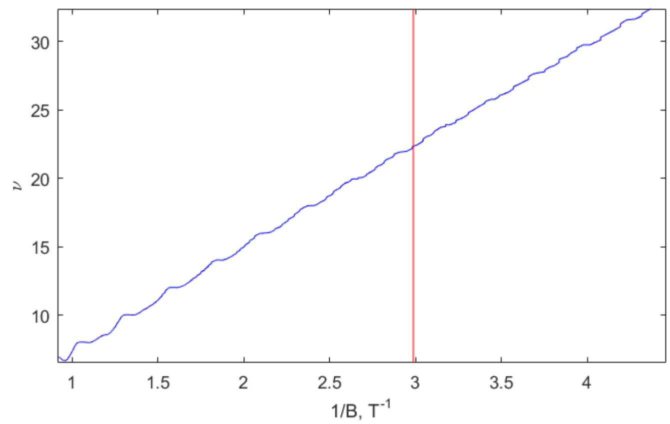
The value of mean free path was calculated using (14) and is equal to $l_{mfp} = 18\ \mu m$. This is also quite reasonable result, considering high mobility value for electrons in this 2DEG which was obtained before as well as previous measurements in this type of semiconductor structures [1].

C. *High-Field Data Analysis*

A second experiment involved Shubnikov-de Haas and Hall resistance measurement in high-field region, see Fig. 8a. In the high-field regime, interesting effects start to occur. In low B-fields, ν as a function of $\frac{1}{B}$ will yield steps of 2 ν at a time as a result of spin. However, at high fields spin separation starts to occur and we can start to resolve steps of 1. By looking at the max value of ν where we still can see the plateaus in Fig. 8b, we can see that there are 22 resolvable LLs in the sample.



(a) High-field range. Red line represents the border between spin separated and non-separated levels.



(b) Lower field range. In this case, red line represents the end of the last resolvable Landau level.

Fig. 8: Filling factor as a function of $\frac{1}{B}$.

V. Conclusions

In this project characteristics of two dimensional electron gas were studied, using the semiconductor structure, represented at Fig. 1. The measurements were done at the temperature range of $13 - 20\ mK$, which allowed us to see a quantum-mechanical behaviour of electrons in magnetic field, quantum Hall effect and Shubnikov-de Haas effect. Comparing the form of obtained resistance curves to the ones plotted in Fig. 6.10 in [2], it is safe to say that measurements were done correctly. There are some differences, however that is because the measurements in [2] were done at $1.13\ K$ and the measurements done for this report, while in a fridge at $13\ mK$, the electrons themselves were around $0.5\ K$ due to being heated by high frequency noise. Later, using the data, we managed to determine the important parameters of electrons in 2DEG such as electron density, mobility, mean free path and Fermi wavelength. All of these values correspond nicely to provided theory and previous experimental results, which approves the validity of the methods used in calculations.

References

[1] Quantum Coherence Lab: Zumbühl Group. Gaas heterostructures and 2d electron gas. *University of Basel*.

[2] John H. Davies. *The Physics of Low-Dimensional Semiconductors: An Introduction*.

[3] M.J. Madou. *Solid-State Physics, Fluidics, and Analytical Techniques in Micro- and Nanotechnology*. CRC Press, 2011.