1 Exercise 17.1-4 Extraction efficiency (p. 690)

b) The critical angle when going from a material with refractive index $n$ to air (vacuum) is given by

$$\Theta_c = \sin^{-1}(1/n) \quad (1)$$

The fraction of light that can be extracted is here denoted $\eta_3$ and is given by the solid angle of the light cone that does not undergo total internal reflection. It is given by

$$\eta_3 = \frac{1}{2}(1 - \sqrt{1 - \frac{1}{n^2}}) \quad (2)$$

The table below shows the critical angles and the extraction efficiencies for the various materials.

<table>
<thead>
<tr>
<th>material</th>
<th>$n$</th>
<th>$\Theta_c$</th>
<th>$\eta_3$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>GaAs</td>
<td>3.5</td>
<td>16</td>
<td>2</td>
</tr>
<tr>
<td>GaN</td>
<td>2.5</td>
<td>24</td>
<td>4</td>
</tr>
<tr>
<td>Polymer</td>
<td>1.5</td>
<td>42</td>
<td>13</td>
</tr>
</tbody>
</table>

c) The combined effect of refraction at the semiconductor-polymer interface and at the polymer-air interface give exactly the same escape cone as the bare semiconductor surface. The enhancement in using a polymer is then either a reduction in Fresnel reflection as discussed in (d) or in the possibility to shape the polymer to a non-planar surface as shown in Figure 17.1-12 (b).

2 Exercise 17.2-5 Amplifier gain and bandwidth (p. 746)

The net gain $\gamma_0(\nu)$ for a (bulk) GaAs amplifier is given by (eq. 17.2-4)

$$\gamma_0(\nu) = \frac{\lambda^2}{8\pi \tau_r} g(\nu) f_g(\nu) \quad (3)$$

At $T = 0$K the Fermi inversion factor $f_g(\nu)$ becomes

$$f_g(\nu) = \begin{cases} 
1 & \nu < E_{fc} - E_{fv} \\
-1 & \text{otherwise}
\end{cases} \quad (4)$$
Figure 1: Gain spectrum

Figure 2: Schematic of the amplifier
We can then plot $\gamma_0(\nu)$ in Figure ??.

A schematic description of the amplifier is given in Figure ??.

This geometry gives a concentration of injected carriers

$$\Delta n = \frac{\tau_i}{\epsilon LDW} = \frac{50 \cdot 10^{-9} \cdot 10^{-3}}{1.6 \cdot 10^{-19} \cdot 2 \cdot 10 \cdot 200 \cdot 10^{-18}} \text{ m}^{-3} = 7.8 \cdot 10^{22} \text{ m}^{-3}$$

(5)

The separation of the quasi-Fermi levels is then (e. g. from exercise 17.1-1)

$$E_{fc} - E_{fv} = E_g + \frac{(3\pi^2)^{2/3}}{2m_r} \Delta n^{2/3} = E_g + 11 \text{ meV}$$

(6)

(Remember the reduced mass $m_r$ defined by $1/m_r = 1/m_e + 1/m_v$)

a) The bandwidth is $E_{fc} - E_{fv} - E_g = 11 \text{ meV}$ or, in frequency, $\Delta \nu = 2.7 \text{ THz}$.

The center frequency is given by

$$\nu_c = \frac{E_g}{\hbar} + \frac{\Delta \nu}{2} = 345 \text{ THz}$$

(7)

The peak net gain $\gamma_0^\text{peak}$ is

$$\gamma_0^\text{peak} = \gamma_0(E_{fc} - E_{fv}) = \frac{\lambda^2}{8\pi \tau_r} \frac{2m_r}{\pi \hbar^2} \sqrt{\frac{\hbar \Delta \nu}{11 \text{ meV}}}$$

(8)

Here $\lambda = \frac{hc}{E_g+11 \text{ meV}} = 867 \text{ nm}$ and thus

$$\gamma_0^\text{peak} = \frac{(867 \cdot 10^{-9})^2}{8\pi \cdot 50 \cdot 10^{-9}} \frac{(2 \cdot 0.06 \cdot 9.1 \cdot 10^{-31})^{3/2}}{\pi(1.05 \cdot 10^{-34})^2} \sqrt{11 \cdot 10^{-3} \cdot 1.6 \cdot 10^{-15} \text{ m}^{-1}} = 260 \text{ cm}^{-1}$$

(9)

b) Number of voice messages: $\Delta \nu/4 \text{ kHz} = 7 \cdot 10^8$ (!)

c) Bit rate: $64 \text{ kbit/s} \cdot 7 \cdot 10^8 = 43 \text{ Tbit/s}$

3 Exercise 17.3-2 Longitudinal modes (p. 747)

As in the previous exercise, the bandwidth is given by the difference between the quasi-Fermi level separation $E_{fc} - E_{fv}$ and the bandgap $E_g$

$$\Delta \nu = (E_{fc} - E_{fv} - E_g)/\hbar = (0.96 - 0.91) \text{ eV}/\hbar = 12 \text{ THz}$$

(10)

The mode spacing is $\nu_F = \frac{c}{2 \nu_m} = \frac{3 \cdot 10^8}{2 \cdot 3.5 \cdot 250 \cdot 10^{-6}} \text{ s}^{-1} = 170 \text{ GHz}$. The maximum number of modes is therefore $\frac{12\text{ THz}}{170\text{ GHz}} = 70$. 

3
4 Exercise 17.3-3 Minimum gain for lasing (p. 747)

The reflection loss coefficient is (eq. 17.3-3)

\[ \alpha_m = \frac{1}{2d} \ln \left( \frac{1}{R_1 R_2} \right) \]  

(11)

To get laser action, the gain coefficient has to be at least as large as the reflection loss coefficient.

The reflectances \( R_1 = R_2 \) are given by the difference in refractive index of the semiconductor and the surrounding air

\[ R_1 = R_2 = \left( \frac{n - 1}{n + 1} \right)^2 = \left( \frac{3.5 - 1}{3.5 + 1} \right)^2 = 0.3 \]  

(12)

Thus

\[ \gamma_0^{\text{lasing}} > \alpha_m = \frac{1}{2 \cdot 500 \cdot 10^{-6}} \ln \left( \frac{1}{0.3^2} \right) \text{ m}^{-1} = 24 \text{ cm}^{-1} \]  

(13)