Coulomb blockade in metallic islands and quantum dots

- Charging energy and chemical potential of a metallic island
- Coulomb blockade and single-electron transistors
- Quantum dots and the constant interaction model
- Finite bias and Coulomb diamonds
- Excited states: transport spectroscopy
- Calculating the current
Different types of quantum dots

- Metallic island

- Gated 2-DEG
Different types of quantum dots

- Semiconducting nanowire

- Carbon nanotube

- Single molecule quantum dot
Metallic island

• Continuous density of states: $\delta E \ll k_B T, \ldots$

• Energy required to add one electron: $E_C \gg k_B T$
Quantum dot: quantized levels

100 nm dot
$E_C = e^2 / C_\Sigma >> \Delta\varepsilon$

30 nm dot
$E_C > \Delta\varepsilon$

10 nm dot
$E_C < \Delta\varepsilon$

Single electron transistor (SET) limit: no energy quantization effects
Quantum dot limit: Quantum effects important

Constant interaction model $\rightarrow$ peak spacing: $e\alpha_g \Delta V_g = \Delta\mu = E_C + \varepsilon_N - \varepsilon_{N-1}$
Finite bias voltage

Small width of resonance: $\Gamma \ll k_B T, E_C$

Current only when at least one $\mu_N$ is between $\mu_L$ and $\mu_R$

Metallic island

In general $\mu_N \propto -e \sum_i \alpha_i V_i$, $\alpha_i = \frac{C_i}{C_\Sigma}$

With $C_L = C_R$, $V_L = -V_R = V/2$

$\mu_N$ is independent of $V$

and the diamonds are straight

$\Delta \mu_N$ in general

Coulomb diamonds (zero current areas)
Finite bias voltage

**Stability diagram**: differential conductance on color scale as a function of gate and bias voltages

\[
\frac{dI(V, V_g)}{dV}
\]
Finite bias voltage

Quantum dot: even diamonds are larger because of spin degeneracy
Finite bias voltage

Carbon nanotube quantum dots: four-fold patter because of additional orbital degeneracy

Leturcq, et. al. (2009)
Finite bias voltage

What are all these other lines?
**Finite bias voltage: excited states**

Energy needed to add electron $N$ to the ground state, provided that the dot is in the $N$-1 electron ground state.

Energy needed to add electron $N$ to the excited state $i$, provided that the dot is in the $N$-1 electron excited state $j$ state.

Can also “tunnel through excited states”

- increased current
- conductance peak

Tunnel spectroscopy of quantum dot levels
**Finite bias voltage: excited states**

Tunneling can lead to occupation of excited states, also with energies larger than $k_B T$.

Current-induced “heating”. To calculate the current, we have to calculate the quantum dot state in nonequilibrium!

Energy needed to add electron $N$ to the ground state, provided that the dot is in the $N$-1 electron ground state.

Energy needed to add electron $N$ to the excited state $i$, provided that the dot is in the $N$-1 electron excited state $j$ state.
Calculating the current

Single electron tunneling: \[ \Gamma \ll k_B T, E_C \]

Rate of electrons tunneling into state \( i \) from lead \( L, R \):

\[ \Gamma_{L,R} \sum_j f(\mu_N^{ij} - \mu_{L,R}) P(j) \]

Rate of tunneling out to lead \( L, R \), leaving the dot in state \( i \):

\[ I = \Gamma_{L,R} \sum_j \left[ 1 - f(\mu_N^{ii} - \mu_{L,R}) \right] P(j) \]

Current:

\[ I = \Gamma_L \sum_{ijN} \left\{ f(\mu_N^{ij} - \mu_{L,R}) - \left[ 1 - f(\mu_N^{ii} - \mu_{L,R}) \right] \right\} P(j) \]

Need occupations! From thermal equilibrium (small \( V \)), or from solving rate (or master) equation
Freaky diamonds and their usefulness

Spectroscopy of vibrational modes in OPV5 molecule

Measuring lifetimes of vibrations in suspended CNTs
Freaky diamonds and their usefulness

Single-molecule magnet in a transport junction?

Can we see the peas in the peapod?
Freaky diamonds and their usefulness

Engineered Majorana fermions in a nanowire?