

Figure 1: Structure of a SET. The current through the island can be controlled via the gate terminal.

#### Quantum Devices: Single electron transistors in nanowires

### 1 Introduction

In this lab we will consider electron transport in nanostructures. More specifically we will look at so called single electron transistors realized in semiconductor nanowires. Nanowires are needle–like objects with a diameter typically less than 100 nm. The one–dimensional nature of the nanowires, and the possibility to use advanced heterostructure technology, make them a very nice system for the study of low–dimensional physics.

The function of a single electron transistor (SET) can be described as follows. Electrons are negatively charged. This means that an energy is required to bring two electrons together. The SET functionality depends directly on this electron–electron repulsion. A SET consists of a small conducting island connected to two electrodes via two tunnel junctions. Due to the electron–electron repulsion and the small size of the island, the electrons are forced to propagate through the SET one at a time. By introducing a third electrode, a gate, the single electron current can be turned on and off. Since the energies involved are quite small the effect is only observed at low temperatures.

### 2 Sample fabrication

The nanowires used in the lab are synthesized using chemical beam epitaxy. The nanowires are grown from gold seed nano-particles. The process is illustrated in figure 2. The low-bandgap material InAs is used for the wire and the high-bandgap material InP inserted for the barriers.

After growth the nanowires are broken off and transferred to a Si substrate covered with a thin silicon oxide ( $\sim 100 \text{ nm}$ ) and contacted with metallic stripes as shown in figure 3a. The metal contacts are fabricated using electron beam lithography and metal evaporation. The metal leads can



Figure 2: (a) Au aerosol nanoparticles are deposited on the substrate. (b) The sample is heated to growth temperature. (c) Growth begins when the source gases are turned on. (Courtesy Mikael Björk)



Figure 3: (a) The wires are placed on a substrate and metal contacts are defined using electron beam lithography. (b) Schematic drawing showing a nanowire with two barriers creating a central island. Typical dimensions for our system are: diameter  $\sim 50$  nm, barrier thickness  $\sim 10$  nm, length of central island  $\sim 100$  nm.

then be connected to the outside world and our measurement setup. The substrate will be used as a gate.

# 3 Theory

We will here give a brief account of the theory behind single electron transistors. What is needed is essentially some electrostatics, so therefore we start by refreshing your memory concerning what capacitance is and the energies involved. Then, we will focus on the coupling between the island and the gate and then include the action of the source-drain bias. During the discussion we try to apply the model to our system, an InAs nanowire with InP barriers, see figure 3b.



Figure 4: The concept of capacitance of a two-conductor system.

#### 3.1 Capacitance

Capacitance is a fundamental concept in electrostatics. Consider two ideal conductors in the vicinity of each other as shown in figure 4. Connecting a dc voltage source between the conductors results in a charge transfer, so that a charge +Q resides on one conductor and a charge -Q on the other. The charge Q can be deduced from the following formula

$$Q = CV_{12} \tag{1}$$

where C is the capacitance of this two-conductor system and  $V_{12}$  is the potential difference between the two conductors. The energy electrostatically stored in this system is

$$U = \frac{Q^2}{2C} \tag{2}$$

The nanowire central island will have a capacitive coupling to the two leads as well as to the underlying substrate. The total capacitance  $C = C_{\Sigma}$ of the island will be the sum of all capacitances it has with respect to the leads and the substrate. To find the energy stored on the island, we substitute  $(Ne)^2$  for  $Q^2$ , where N is the number of electrons on the island.

#### 3.2 Coulomb Oscillations

The bandstructure of our nanowire device is shown in figure 5a. At low temperatures, the nanowire leads are filled with electrons up to the chemical potential,  $\mu_s$  and  $\mu_d$ , where the indices s and d denotes source and drain. These are connected to the outer world and applying a source-drain bias induces a shift between the chemical potentials,  $V_{sd} = (\mu_s - \mu_d)/e$ . Electrons

can only flow through the device if the island has a energy level in between  $\mu_s$  and  $\mu_d$ .

The chemical potential of the island defined by the energy an electron must have to be able to enter the system is given by,  $\mu_{island}(N) = U(N) - U(N-1)$ , with U(N) being the total electrostatic ground state energy of the island. We will model the electron-electron interaction between electrons on the island and electrons in the electrodes by a capacitance. The island has three capacitances,  $C_s$ ,  $C_d$  and  $C_g$ , originating from the source, drain and gate capacitance to the island. The sum of these is the total capacitance of the island,  $C_{\Sigma}$ . We will also assume that  $C_{\Sigma}$  does not depend on N. Inserting equation 2 into the definition of  $\mu_{island}$  and adding the continuous action of  $V_g$  leads us to an expression for the chemical potential of the island;

$$\mu_{island}(N) = \frac{(N-1/2)e^2}{C_{\Sigma}} - e\frac{C_g}{C_{\Sigma}}V_g \tag{3}$$

The first term is fixed for a fixed electron number on the island, whereas the second term changes  $\mu_{island}$  continuously with  $V_g$ . This continuous shift can be seen as shifting the energy levels on the island as indicated by the arrows in figure 5a. At a fixed  $V_g$  the electron number, N, on the island is the largest integer for which  $\mu_{island}(N) < \mu_s, \mu_d$ .

If a small  $V_{sd}$  is applied,  $\mu_s$  and  $\mu_d$  are shifted relative to each other and we open a transport window,  $V_{sd} = (\mu_s - \mu_d)/e$ , seen in figure 5a. As mentioned earlier, electrons can only tunnel through the device if an energy level is in the transport window, as is the case in figure 5a. Making  $V_g$ more positive lowers  $\mu_{island}$  and eventually the energy level in the transport window will fall below  $\mu_d$  and the electron occupying that level has no free states in the drain to tunnel out to. The electron is now trapped on the island and N is increased by 1 in equation 3. This trapping causes the current to drop because we no longer have an energy level in the transport window.

In order to bring down the next level into the transport window we need to change the chemical potential of the dot by

$$\mu_{island}(N+1) - \mu_{island}(N) = \frac{e^2}{C_{\Sigma}} \equiv E_c \tag{4}$$

The energy in equation 4 is called the charging energy and is denoted  $E_c$ . We can add this energy to the island by changing  $V_g$  and we see that in order for the second term in equation 3 to equal  $\frac{e^2}{C_{\Sigma}}$ ,  $V_g C_g$  must equal e. So, by changing  $V_g$  an amount  $e/C_g$  we move from one specific energy level in the transport window to the next.

Every time there is an energy level in the transport window the current through the device is non-zero and otherwise it is zero. This current/no current switching leads to current peaks when we sweep  $V_g$ , as seen in figure 5b. This is called Coulomb oscillations.



Figure 5: Effect of the gate (a) Changing the gate potential will effectively move the states on the island up and down in energy. (b) So called Coulomb oscillations in the current through the SET.



Figure 6: (a) Zero bias, the coulomb energy required to tunnel into the island creates a blockade region and there is no available states for the electrons to tunnel into. The current through the device is zero. (b)With enough bias we reach the threshold voltage where the state inside the island becomes available for tunnelling. Electrons can tunnel through the device one by one from left to right.

#### 3.3 Effect of the source–drain bias

In figure 6a the island is in a blocked state, i.e. we have no energy levels in the transport window. This situation corresponds to a valley in the Coulomb oscillations, seen in figure 5b. As we described in the previous section we can change  $V_g$  in order to get an energy level in the transport window and then get current through the device. But since we have more electrodes to the island we could also use these to add energy to the system.

Let us now apply a small  $V_{sd}$  to a situation depicted in figure 6a. If  $V_{sd}$  is small enough, there will not be any current through the device since there is no energy level in the transport window. We need to apply a large  $V_{sd}$ , as shown in figure 6b in order to have the next level in the transport window. This will lead to current-voltage characteristics similar to the solid line in

figure 7.

If we apply a finite  $V_{sd}$  in the situation depicted in figure 5a, where there already is an energy level in the transport window, we get an increase in current similar to the dotted line in figure 7.

Now consider the case that we sweep both  $V_{sd}$  and  $V_g$  simultaneously. Can you visualize how the 3D plot  $I_{sd}$  Vs.  $(V_{sd}, V_g)$  would look like ?

### 4 Preparation questions

You should prepare answers to these questions *before* the lab session. We will begin the lab by discussing them.

- 1. If we model the capacitance between the island and one of the leads using a parallel plate capacitor model, with the barrier as the insulating dielectric, what is the capacitance? Use 50 nm for the wire diameter and 7 nm for the barrier thickness. The relative dielectric constant of InP is 12.5.
- 2. If we add an electron to the central island on the nanowire discussed in (1), what is the charging energy? Use  $C_1 = C_2 = C_g$
- 3. Do you think it is a good approximation to use the same value for  $C_g$  as for  $C_1$  and  $C_2$ ? Why / why not?
- 4. What is the thermal energy of an electron at liquid helium temperature (4.2K) and at room temperature? How does it relate to the energy obtained in question (2)? Is it necessary to measure the I-V characteristics at liquid helium temperature?
- 5. We will perform measurements where we sweep the source-drain bias and gate voltages. What is the smallest bias voltage step size which it makes sense to use considering that the sample is at 4.2K?
- 6. Typical measured currents will be in the order of pA, what tunnelling frequency does it correspond to, i.e. how often does an electron tunnel in and out of the island?

### 5 Measurement setup

We will use a voltage source with multiple outputs to apply the source– drain voltage,  $V_{sd}$ , and the gate voltage  $V_g$ . The voltages are controlled by a computer which also reads the current and plots the acquired data. Since we are measuring currents in the range of pA we also need a current amplifier. The sample is mounted on a measurement stick and placed in a liquid helium



Figure 7: The coulomb charging energy will result in a blockade region in the IV curve as shown by the solid line. The dotted line shows an ohmic IV–curve.

container that keeps the sample temperature at 4.2 K. The setup is sketched in figure 8.

Since the nanowires are very sensitive the sample will be in the helium dewar when you start the lab exercise.

## 6 Measurements

The following measurements will be performed during the lab exercise.

- 1. Measure Coulomb oscillations by applying  $V_{sd} = 1$ mV and sweep  $V_g$ . Remember to set the current amplifier in a suitable range. Redo the sweep for a  $V_{sd}$  you think is suitable.
- 2. Measure the  $I_{sd}$  vs.  $V_{sd}$  characteristics in the Coulomb blockade and the non–Coulomb blockade state.
- 3. Measure a stability diagram (Sweep  $V_g$  and  $V_{sd}$  simultaneously and measure  $I_{sd}$ ).

## 7 Lab report

Think about the following when writing your lab report

- Introduce the concepts necessary to understand the measurements and the physics.
- Make sure there is a strong connection between theory and your data.



Figure 8: Schematic drawing of the measurement setup.

• Present and discuss your results and make comparisons to the preparation questions.

Make sure you address the following issues when writing. Work the answers into your text instead of making an questions and answers list.

- What was the periodicity of the gate oscillations?
- How large was the gate capacitance?
- How large was the charging energy?
- How large was the total capacitance of the island to ground?
- What is the resistance of the SET in the conducting state?
- If the temperature is raised from liquid helium temperature, at what temperature do you expect the device to stop working? (No exact figures, just common sense!)
- Comment on the lab: Was it clear and understandable? Could we improve something for the next year?

## 8 Further reading

Transport in nanostructures [1] is a thorough reference book treating SET and many other subjects in low dimensional transport physics. Few-electron quantum dots [2] is an excellent review articles on the topic.

Our department also have several publications on electron transport in nanowire heterostructures. See e.g. Single-electron transistors in heterostructure nanowires [3], Few-Electron Quantum Dots in Nanowires [4] and Nanowire resonant tunnelling diodes [5].

## References

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